Mechanics

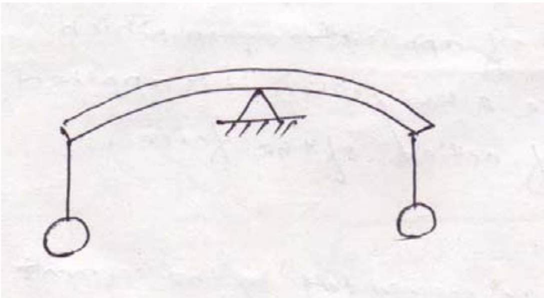
It is defined as that branch of science, which describes and predicts the conditions of rest or motion of bodies under the action of forces. Engineering mechanics applies the principle of mechanics to design, taking into account the effects of forces.

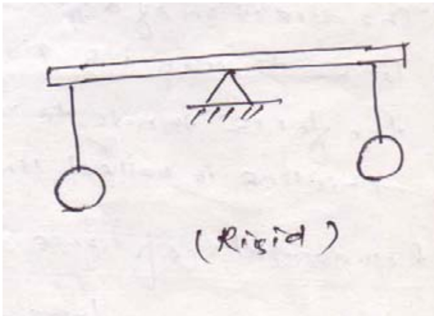
Statics

Statics deal with the condition of equilibrium of bodies acted upon by forces.

Rigid body

A rigid body is defined as a definite quantity of matter, the parts of which are fixed in position relative to each other. Physical bodies are never absolutely but deform slightly under the action of loads. If the deformation is negligible as compared to its size, the body is termed as rigid.



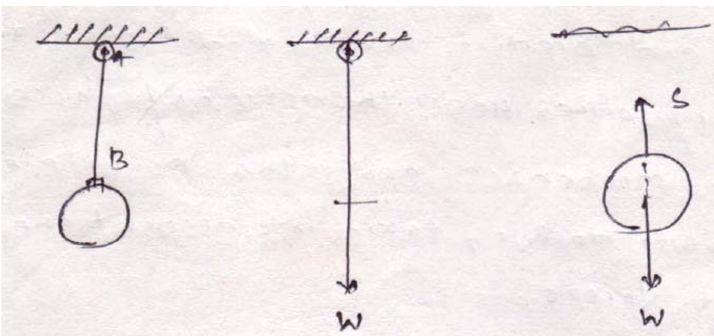


Force

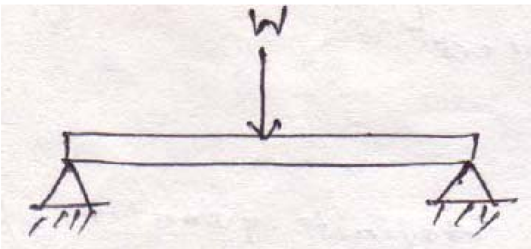
Force may be defined as any action that tends to change the state of rest or motion of a body to which it is applied.

The three quantities required to completely define force are called its specification or characteristics. So the characteristics of a force are:

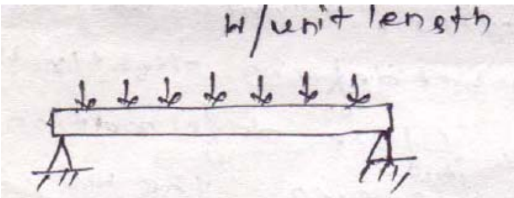
1. Magnitude
2. Point of application
3. Direction of application



Concentrated force/point load



Distributed force

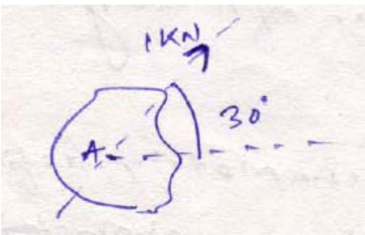


Line of action of force

The direction of a force is the direction, along a straight line through its point of application in which the force tends to move a body when it is applied. This line is called line of action of force.

Representation of force

Graphically a force may be represented by the segment of a straight line.

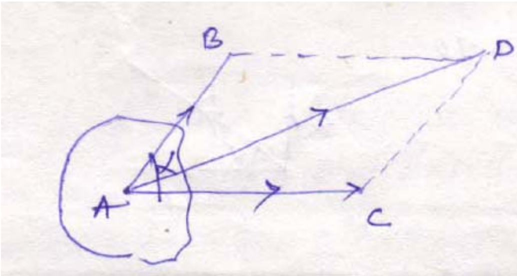


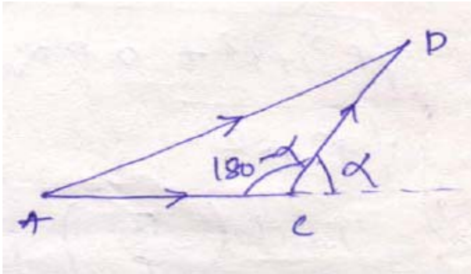
Composition of two forces

The reduction of a given system of forces to the simplest system that will be its equivalent is called the problem of composition of forces.

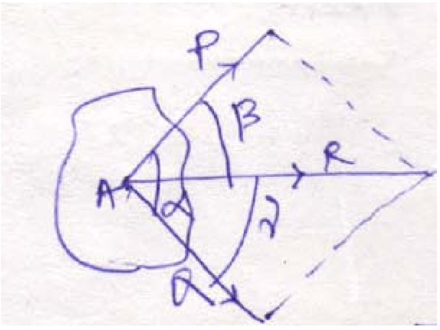
Parallelogram law

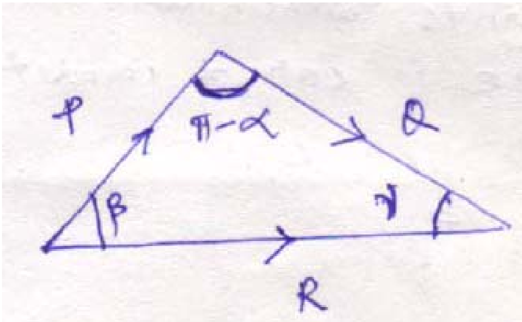
If two forces represented by vectors AB and AC acting under an angle α are applied to a body at point A. Their action is equivalent to the action of one force, represented by vector AD, obtained as the diagonal of the parallelogram constructed on the vectors AB and AC directed as shown in the figure.





Force AD is called the resultant of AB and AC and the forces are called its components.





$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos\alpha\right)}$$

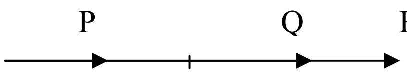
Now applying triangle law

$$\frac{P}{Sin\gamma} = \frac{Q}{Sin\beta} = \frac{R}{Sin(\pi - \alpha)}$$

Special cases

Case-I: If α = 0˚

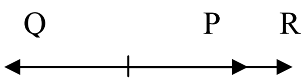
$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos0^\circ\right)} = \sqrt{(P+Q)^2} = P + Q$$



R = P+Q

Case- II: If α = 180˚

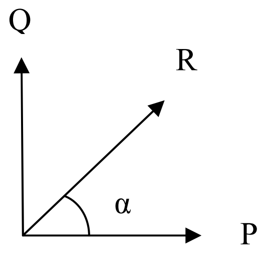
$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos180^\circ\right)} = \sqrt{(P^2 + Q^2 - 2PQ)} = \sqrt{(P-Q)^2} = P - Q$$



Case-III: If $\alpha = 90^\circ$

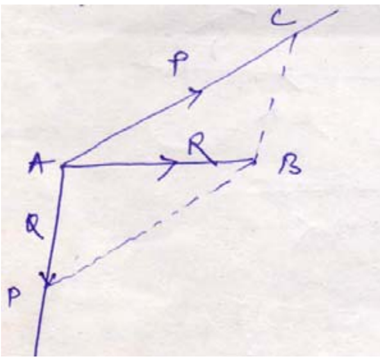
$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos90^\circ\right)} = \sqrt{P^2 + Q^2}$$

$\alpha = \tan^{-1}(Q/P)$



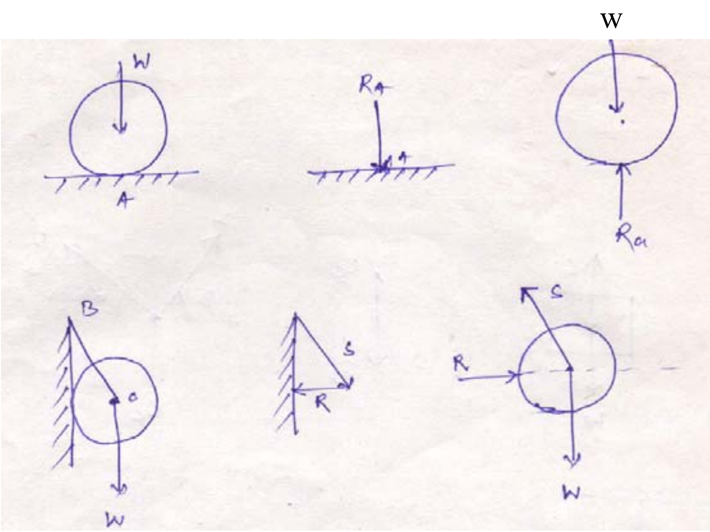
Resolution of a force

The replacement of a single force by a several components which will be equivalent in action to the given force is called resolution of a force.



Action and reaction

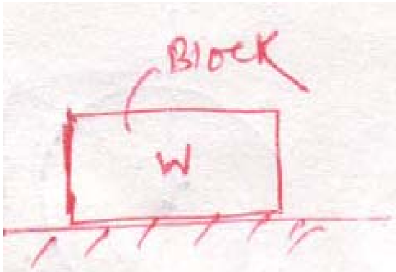
Often bodies in equilibrium are constrained to investigate the conditions.

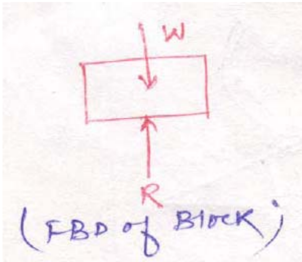


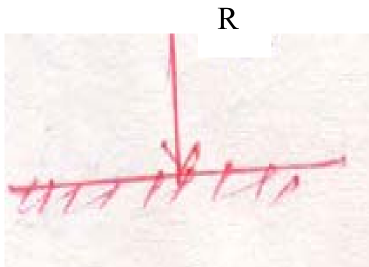
Free body diagram

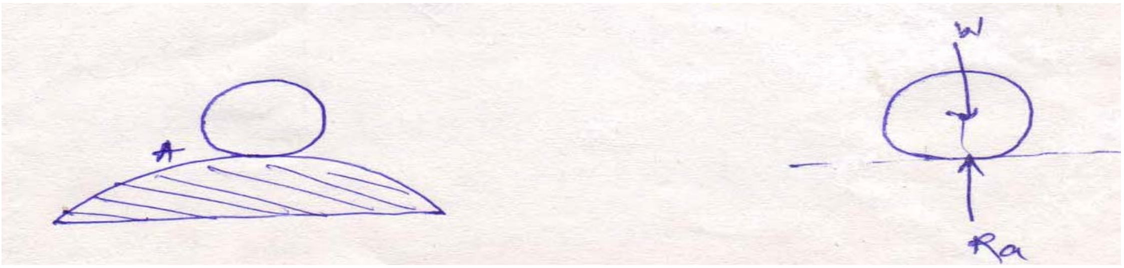
Free body diagram is necessary to investigate the condition of equilibrium of a body or system. While drawing the free body diagram all the supports of the body are removed and replaced with the reaction forces acting on it.

1. Draw the free body diagrams of the following figures.

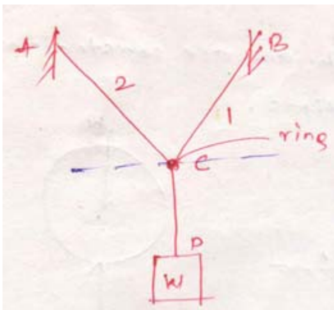


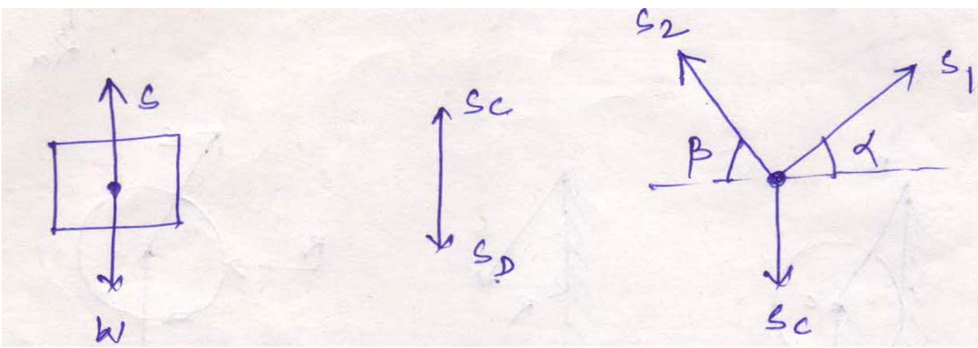




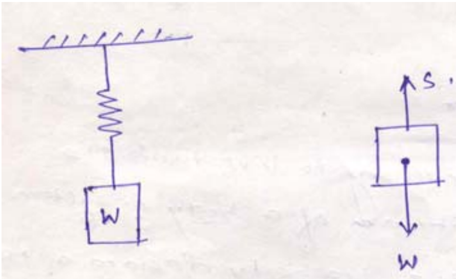


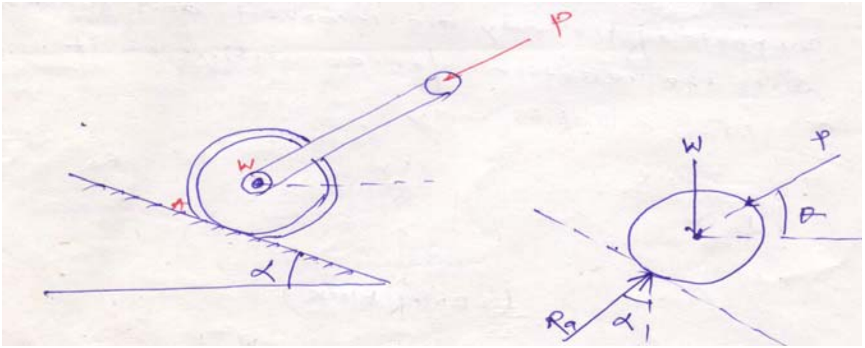
2. Draw the free body diagram of the body, the string CD and the ring.





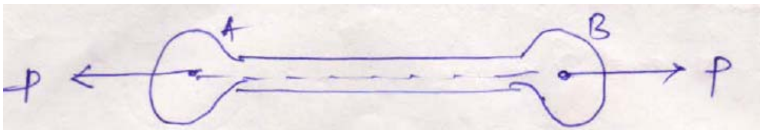
3. Draw the free body diagram of the following figures.



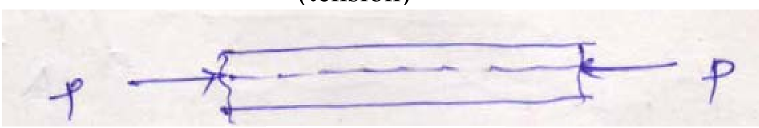


Equilibrium of colinear forces:

Equllibrium law: Two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action.



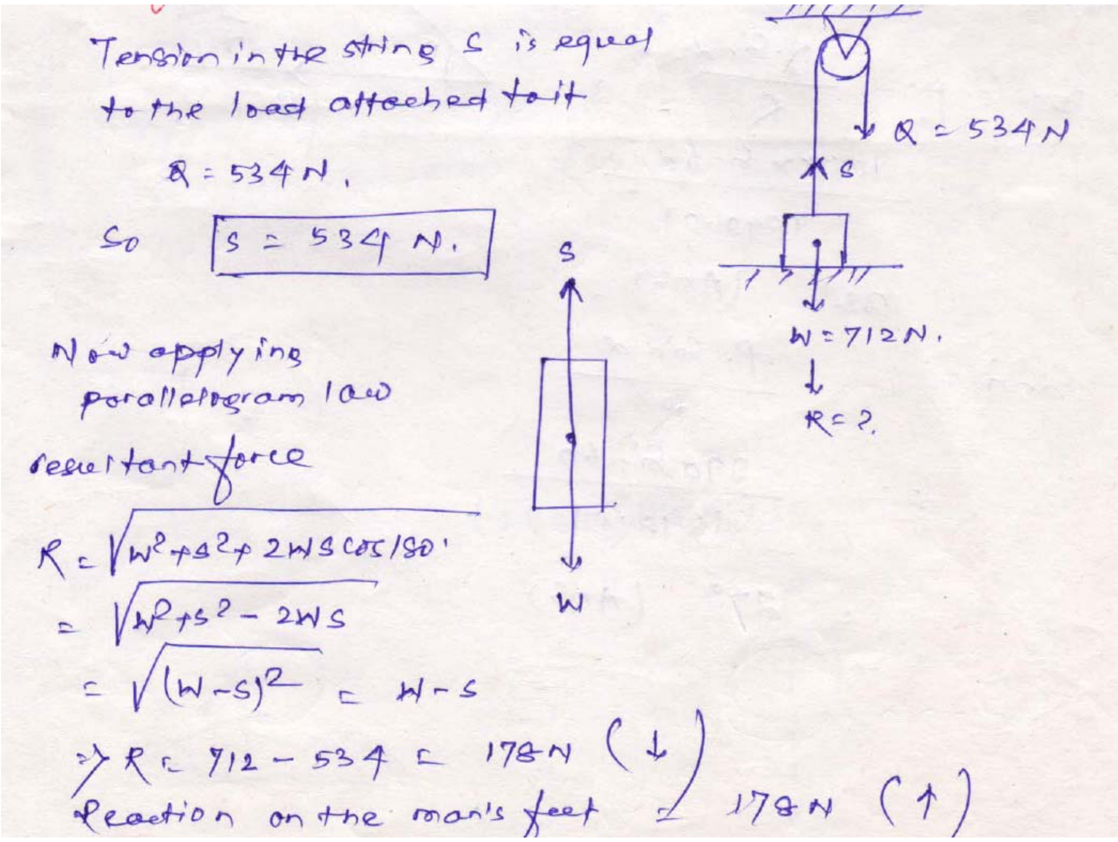
(tension)



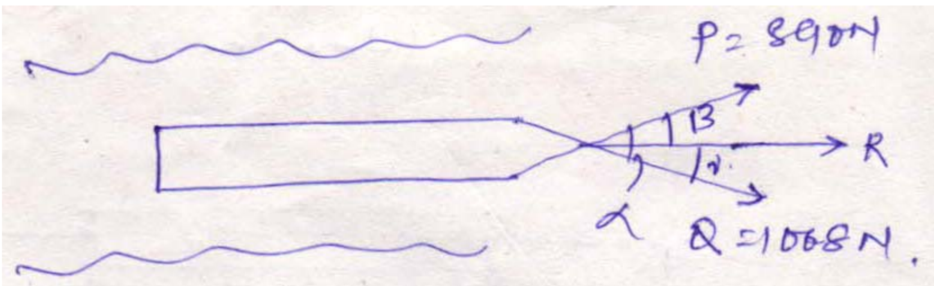
(compression)

Superposition and transmissibility

Problem 1: A man of weight W = 712 N holds one end of a rope that passes over a pulley vertically above his head and to the other end of which is attached a weight Q = 534 N. Find the force with which the man's feet press against the floor.



Problem 2: A boat is moved uniformly along a canal by two horses pulling with forces P = 890 N and Q = 1068 N acting under an angle α = 60˚. Determine the magnitude of the resultant pull on the boat and the angles β and ν.



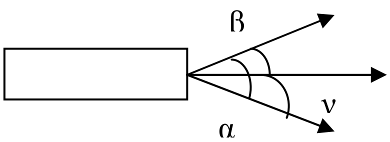
P = 890 N, α = 60˚

Q = 1068 N

$$R = \sqrt{(P^2 + Q^2 + 2PQ\cos\alpha)}$$

$$= \sqrt{(890^2 + 1068^2 + 2\times 890\times 1068\times 0.5)}$$

$$= 1698.01N$$

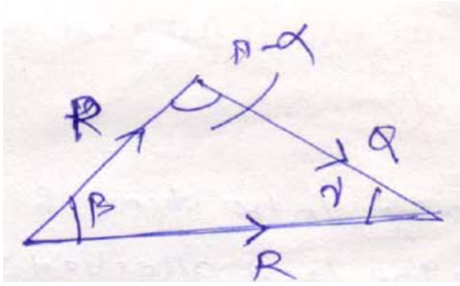


$$\frac{Q}{\sin\beta} = \frac{P}{\sin\nu} = \frac{R}{\sin(\pi-\alpha)}$$

$$\sin\beta = \frac{Q\sin\alpha}{R}$$

$$= \frac{1068\times\sin 60^\circ}{1698.01}$$

$$= 33^\circ$$



$$\sin\nu = \frac{P\sin\alpha}{R}$$

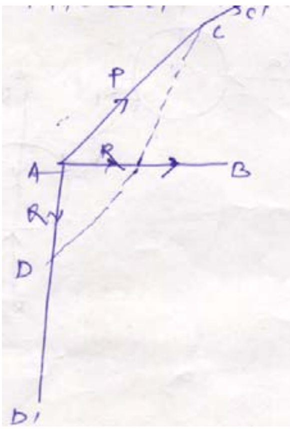
$$= \frac{890\times\sin 60^\circ}{1698.01}$$

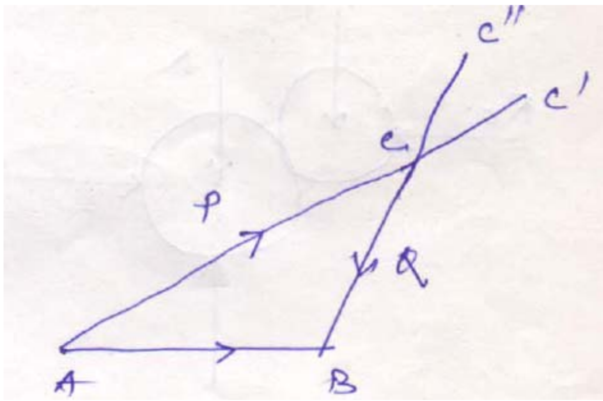
$$= 27^\circ$$

<u>Resolution of a force</u>

Replacement of a single force by several components which will be equivalent in action to the given force is called the problem of resolution of a force.

By using parallelogram law, a single force R can be resolved into two components P and Q intersecting at a point on its line of action.





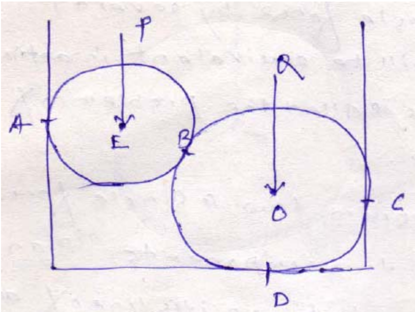
<u>Equilibrium of collinear forces:</u>

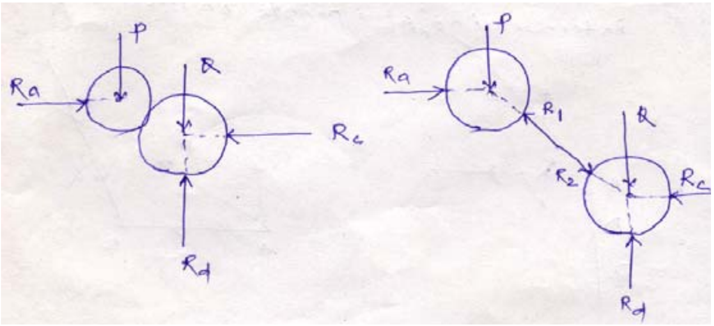
Equilibrium law: Two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action.

Law of superposition

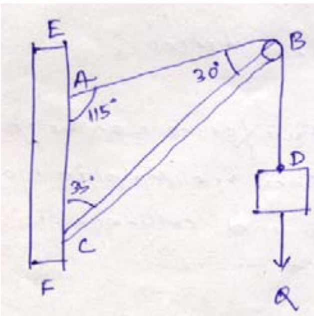
The action of a given system of forces on a rigid body will no way be changed if we add to or subtract from them another system of forces in equllibrium.

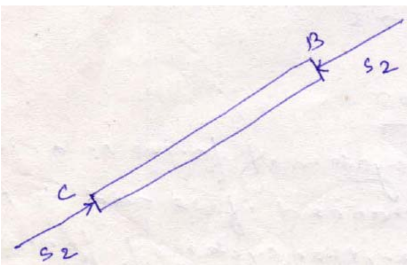
Problem 3: Two spheres of weight P and Q rest inside a hollow cylinder which is resting on a horizontal force. Draw the free body diagram of both the spheres, together and separately.

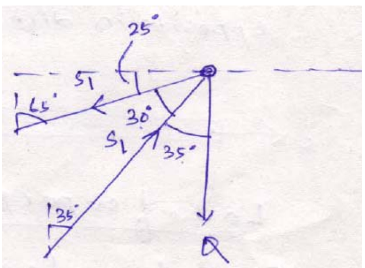




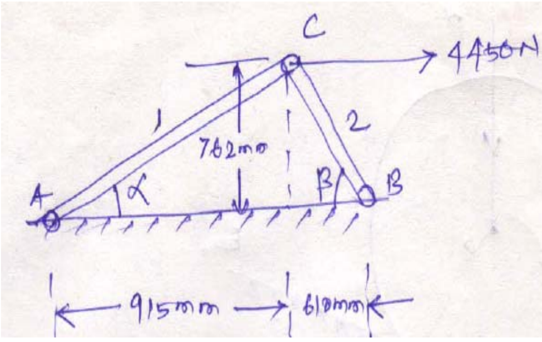
Problem 4: Draw the free body diagram of the figure shown below.

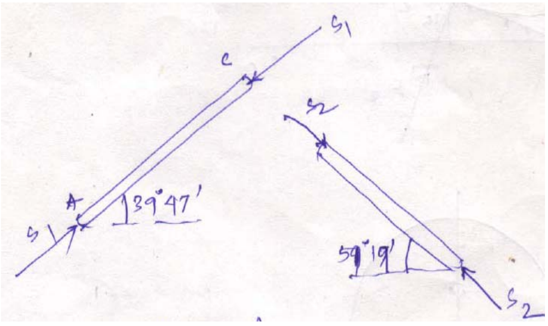




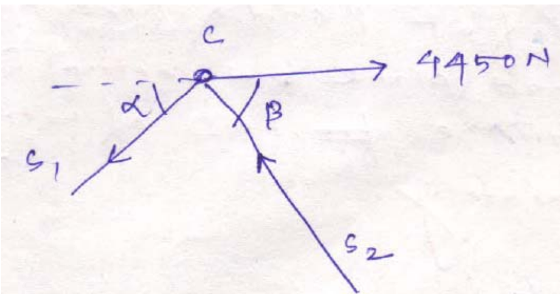


Problem 5: Determine the angles α and β shown in the figure.

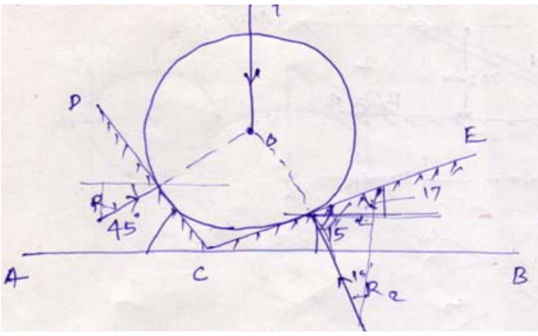


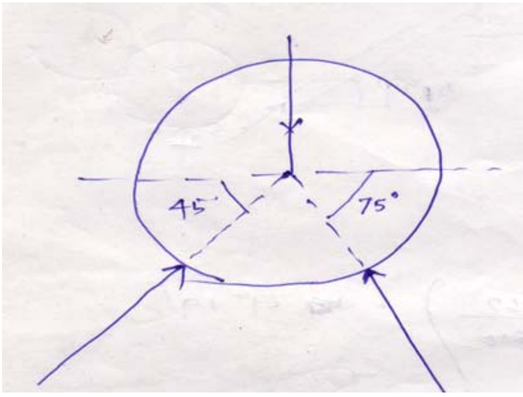


$$\alpha = \tan^{-1}\left(\frac{762}{915}\right)$$
$$= 39^{\circ}47^{'}$$
$$\beta = \tan^{-1}\left(\frac{762}{610}\right)$$
$$= 51^{\circ}19^{'}$$

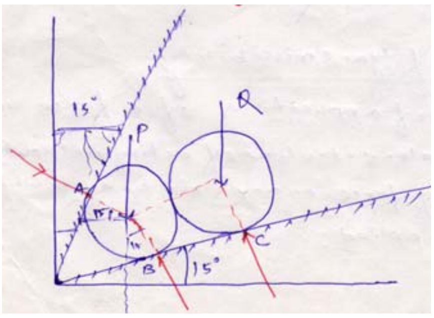


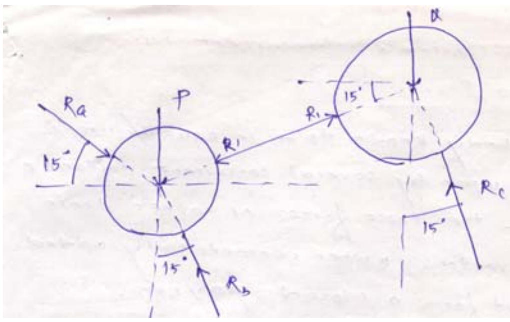
Problem 6: Find the reactions R_1 and R_2.



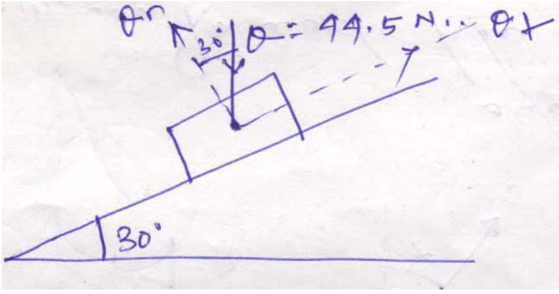


Problem 7: Two rollers of weight P and Q are supported by an inclined plane and vertical walls as shown in the figure. Draw the free body diagram of both the rollers separately.

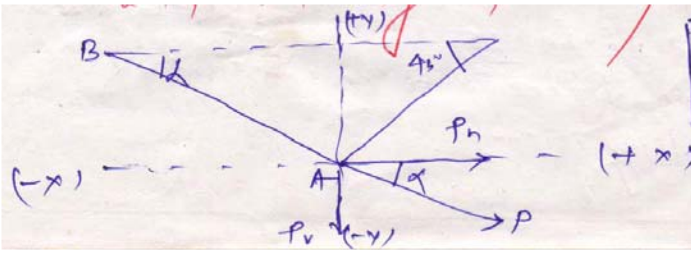




Problem 8: Find θ_n and θ_t in the following figure.



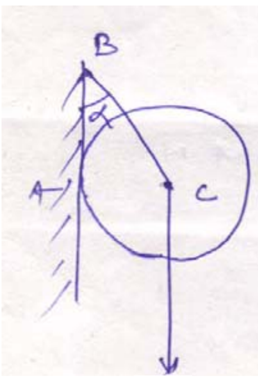
Problem 9: For the particular position shown in the figure, the connecting rod BA of an engine exert a force of P = 2225 N on the crank pin at A. Resolve this force into two rectangular components P_h and P_v horizontally and vertically respectively at A.



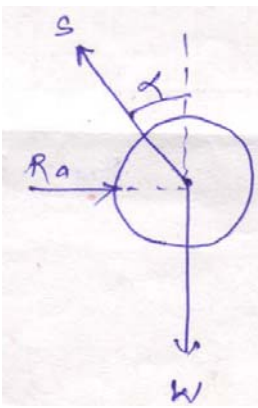
P_h = 2081.4 N
P_v = 786.5 N

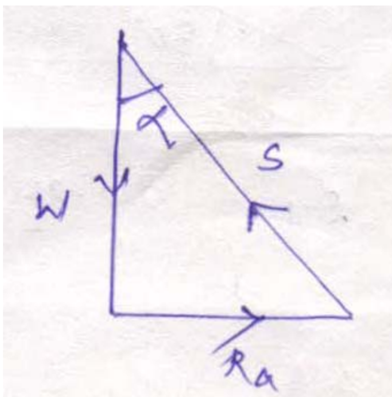
Equilibrium of concurrent forces in a plane

- If a body known to be in equilibrium is acted upon by several concurrent, coplanar forces, then these forces or rather their free vectors, when geometrically added must form a closed polygon.
- This system represents the condition of equilibrium for any system of concurrent forces in a plane.



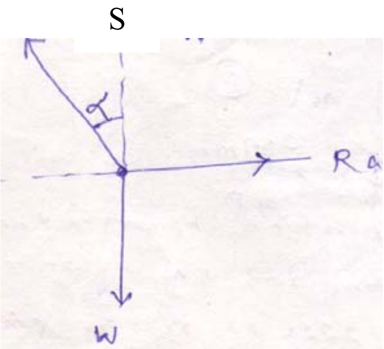
W





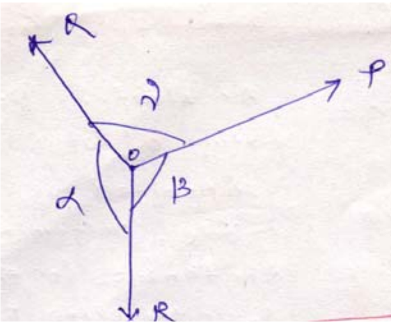
$R_a = w\tan\alpha$

$S = w\sec\alpha$

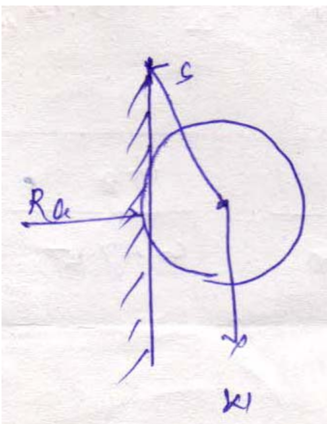


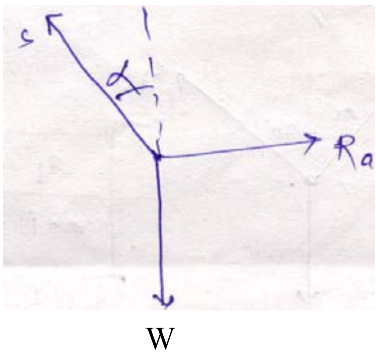
Lami’s theorem

If three concurrent forces are acting on a body kept in an equllibrium, then each force is proportional to the sine of angle between the other two forces and the constant of proportionality is same.



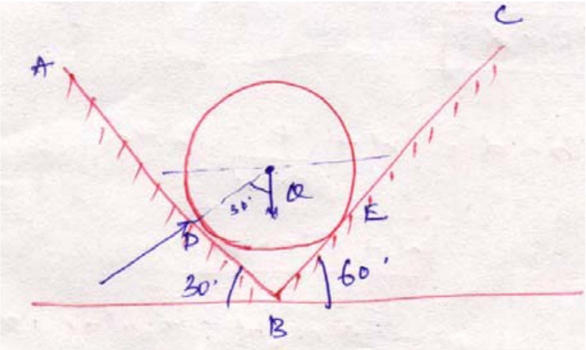
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\upsilon}$$

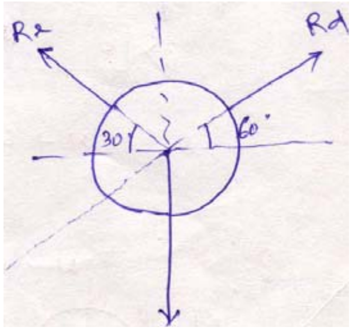




$$\frac{S}{\sin 90} = \frac{R_a}{\sin(180-\alpha)} = \frac{W}{\sin(90+\alpha)}$$

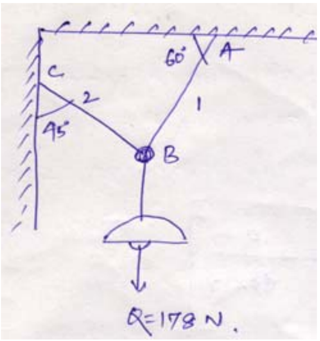
Problem: A ball of weight Q = 53.4N rest in a right angled trough as shown in figure. Determine the forces exerted on the sides of the trough at D and E if all the surfaces are perfectly smooth.

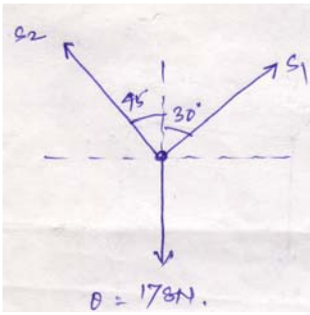




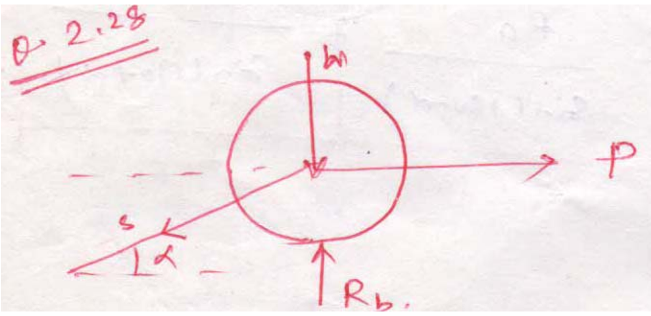
W

Problem: An electric light fixture of weight Q = 178 N is supported as shown in figure. Determine the tensile forces S_1 and S_2 in the wires BA and BC, if their angles of inclination are given.





$$\frac{S_1}{\sin 135} = \frac{S_2}{\sin 150} = \frac{178}{\sin 75}$$

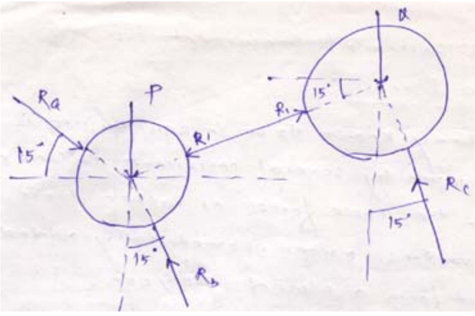


$$S_1 \cos\alpha = P$$

S = Psecα

$$R_b = W + S\sin\alpha$$
$$= W + \frac{P}{\cos\alpha} \times \sin\alpha$$
$$= W + P\tan\alpha$$

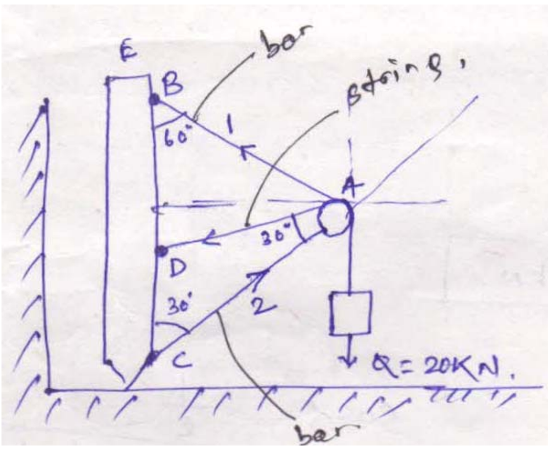
Problem: A right circular roller of weight W rests on a smooth horizontal plane and is held in position by an inclined bar AC. Find the tensions in the bar AC and vertical reaction R_b if there is also a horizontal force P is active.

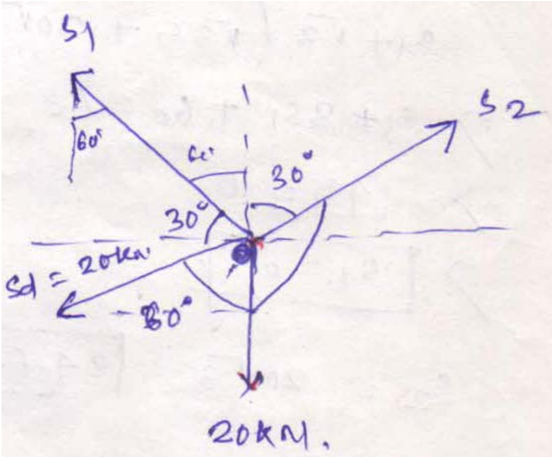


Theory of transmissiibility of a force:

The point of application of a force may be transmitted along its line of action without changing the effect of force on any rigid body to which it may be applied.

Problem:





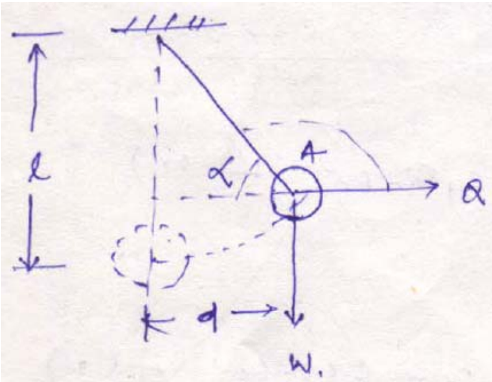
$$\sum X = 0$$
$$S_1 \cos 30 + 20 \sin 60 = S_2 \sin 30$$
$$\frac{\sqrt{3}}{2} S_1 + 20\frac{\sqrt{3}}{2} = \frac{S_2}{2}$$
$$\frac{S_2}{2} = \frac{\sqrt{3}}{2} S_1 + 10\sqrt{3}$$
$$S_2 = \sqrt{3} S_1 + 20\sqrt{3} \qquad (1)$$

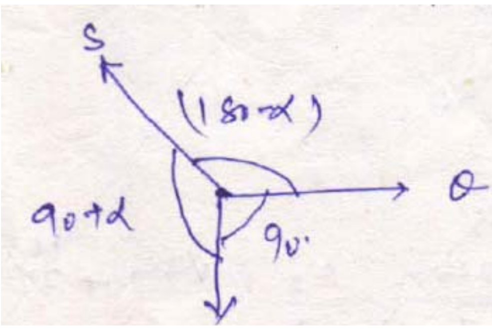
$$\sum Y = 0$$
$$S_1 \sin 30 + S_2 \cos 30 = S_d \cos 60 + 20$$
$$\frac{S_1}{2} + S_2 \frac{\sqrt{3}}{2} = \frac{20}{2} + 20$$
$$\frac{S_1}{2} + \frac{\sqrt{3}}{2} S_2 = 30$$
$$S_1 + \sqrt{3} S_2 = 60 \qquad (2)$$

Substituting the value of S_2 in Eq.2, we get

$$S_1 + \sqrt{3}\left(\sqrt{3} S_1 + 20\sqrt{3}\right) = 60$$
$$S_1 + 3S_1 + 60 = 60$$
$$4S_1 = 0$$
$$S_1 = 0KN$$
$$S_2 = 20\sqrt{3} = 34.64KN$$

Problem: A ball of weight W is suspended from a string of length l and is pulled by a horizontal force Q. The weight is displaced by a distance d from the vertical position as shown in Figure. Determine the angle α, forces Q and tension in the string S in the displaced position.





W

$$\cos\alpha = \frac{d}{l}$$

$$\alpha = \cos^{-1}\left(\frac{d}{l}\right)$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\Rightarrow \sin\alpha = \sqrt{(1-\cos^2\alpha)}$$

$$= \sqrt{1-\frac{d^2}{l^2}}$$

$$= \frac{1}{l}\sqrt{l^2-d^2}$$

Applying Lami's theorem,

$$\frac{S}{\sin 90} = \frac{Q}{\sin(90+\alpha)} = \frac{W}{\sin(180-\alpha)}$$

$$\frac{Q}{\sin(90+\alpha)} = \frac{W}{\sin(180-\alpha)}$$

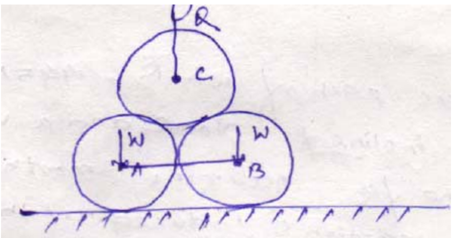
$$\Rightarrow Q = \frac{W\cos\alpha}{\sin\alpha} = \frac{W\left(\frac{d}{l}\right)}{\frac{1}{l}\sqrt{l^2-d^2}}$$

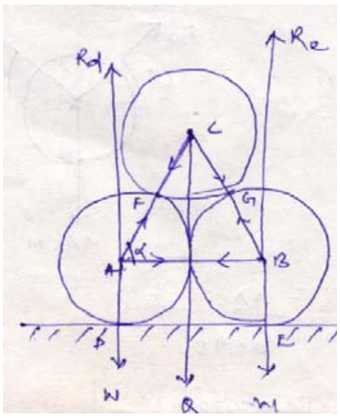
$$\Rightarrow Q = \frac{Wd}{\sqrt{l^2-d^2}}$$

$$S = \frac{W}{\sin\alpha} = \frac{W}{\frac{1}{l}\sqrt{l^2-d^2}}$$

$$= \frac{Wl}{\sqrt{l^2-d^2}}$$

Problem: Two smooth circular cylinders each of weight W = 445 N and radius r = 152 mm are connected at their centres by a string AB of length l = 406 mm and rest upon a horizontal plane, supporting above them a third cylinder of weight Q = 890 N and radius r = 152 mm. Find the forces in the string and the pressures produced on the floor at the point of contact.

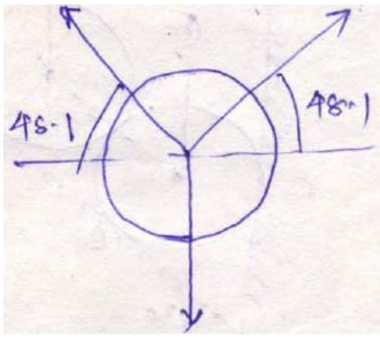




$$\cos\alpha = \frac{203}{304}$$

$$\Rightarrow \alpha = 48.1^\circ$$

R_g R_f



Q

$$\frac{R_g}{\sin 138.1} = \frac{R_e}{\sin 138.1} = \frac{Q}{83.8}$$

$$\Rightarrow R_g = R_e = 597.86N$$

Resolving horizontally

$$\sum X = 0$$

$$S = R_f \cos 48.1$$

$$= 597.86 \cos 48.1$$

$$= 399.27N$$

Resolving vertically

$$\sum Y = 0$$

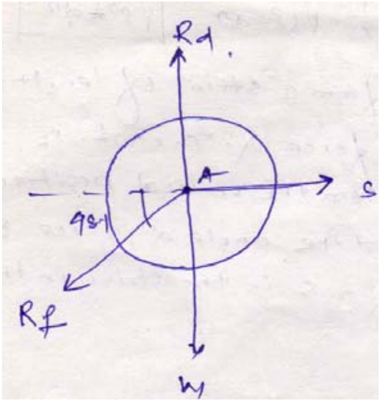
$$R_d = W + R_f \sin 48.1$$

$$= 445 + 597.86 \sin 48.1$$

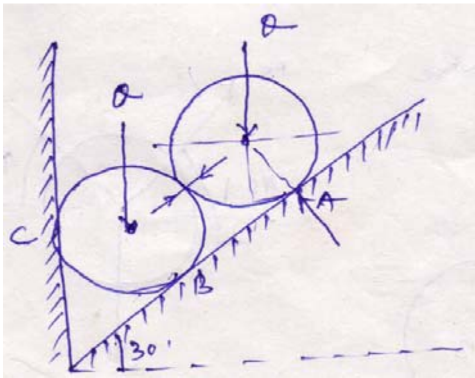
$$= 890N$$

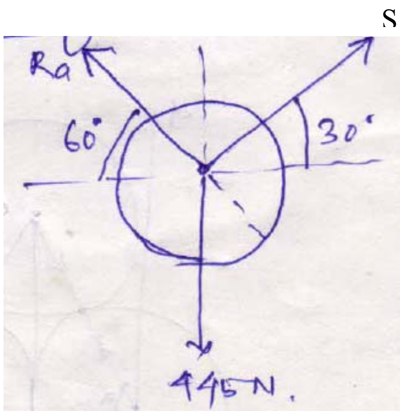
$$R_e = 890N$$

$$S = 399.27N$$



Problem: Two identical rollers each of weight Q = 445 N are supported by an inclined plane and a vertical wall as shown in the figure. Assuming smooth surfaces, find the reactions induced at the points of support A, B and C.





$$\frac{R_a}{\sin 120} = \frac{S}{\sin 150} = \frac{445}{\sin 90}$$

$$\Rightarrow R_a = 385.38N$$

$$\Rightarrow S = 222.5N$$

Resolving vertically

$$\sum Y = 0$$

$$R_b \cos 60 = 445 + S \sin 30$$

$$\Rightarrow R_b \frac{\sqrt{3}}{2} = 445 + \frac{222.5}{2}$$

$$\Rightarrow R_b = 642.302N$$

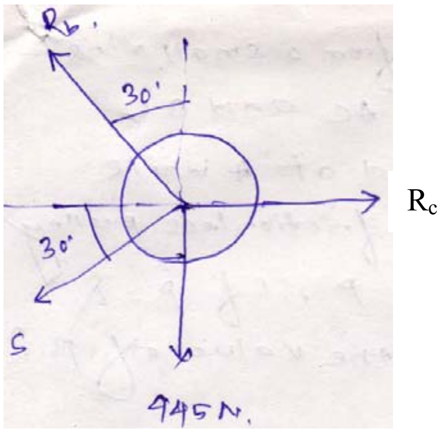
Resolving horizontally

$$\sum X = 0$$

$$R_c = R_b \sin 30 + S \cos 30$$

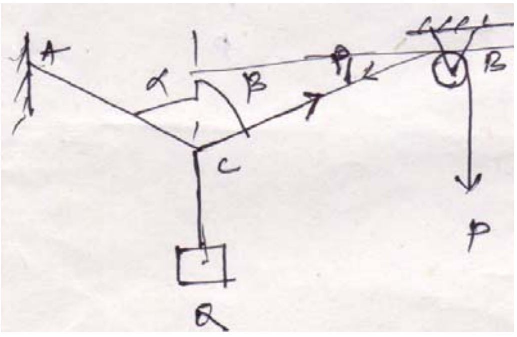
$$\Rightarrow 642.302 \sin 30 + 222.5 \cos 30$$

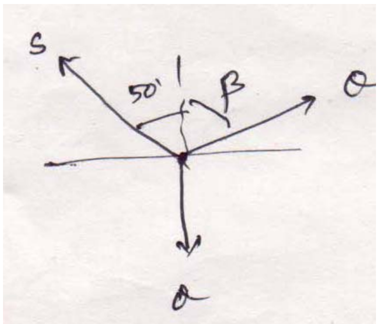
$$\Rightarrow R_c = 513.84N$$



Problem:

A weight Q is suspended from a small ring C supported by two cords AC and BC. The cord AC is fastened at A while cord BC passes over a frictionless pulley at B and carries a weight P. If P = Q and α = 50°, find the value of β.





Resolving horizontally

$$\sum X = 0$$

$$S \sin 50 = Q \sin \beta \quad (1)$$

Resolving vertically

$$\sum Y = 0$$

$$S \cos 50 + Q \sin \beta = Q$$

$$\Rightarrow S \cos 50 = Q(1 - \cos \beta)$$

Putting the value of S from Eq. 1, we get

$$S\cos 50 + Q\sin\beta = Q$$

$$\Rightarrow S\cos 50 = Q(1-\cos\beta)$$

$$\Rightarrow Q\frac{\sin\beta}{\sin 50}\cos 50 = Q(1-\cos\beta)$$

$$\Rightarrow \cot 50 = \frac{1-\cos\beta}{\sin\beta}$$

$$\Rightarrow 0.839\sin\beta = 1-\cos\beta$$

Squaring both sides,

$$0.703\sin^2\beta = 1+\cos^2\beta - 2\cos\beta$$

$$0.703(1-\cos^2\beta) = 1+\cos^2\beta - 2\cos\beta$$

$$0.703 - 0.703\cos^2\beta = 1+\cos^2\beta - 2\cos\beta$$

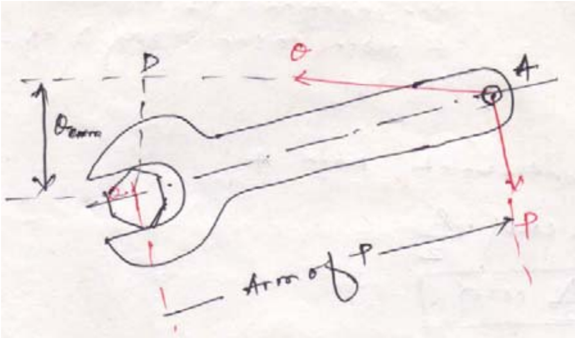
$$\Rightarrow 1.703\cos^2\beta - 2\cos\beta + 0.297 = 0$$

$$\Rightarrow \cos^2\beta - 1.174\cos\beta + 0.297 = 0$$

$$\Rightarrow \beta = 63.13^\circ$$

Method of moments

Moment of a force with respect to a point:



- Considering wrench subjected to two forces P and Q of equal magnitude. It is evident that force P will be more effective compared to Q, though they are of equal magnitude.
- The effectiveness of the force as regards it is the tendency to produce rotation of a body about a fixed point is called the moment of the force with respect to that point.
- Moment = Magnitude of the force × Perpendicular distance of the line of action of force.
- Point O is called moment centre and the perpendicular distance (i.e. OD) is called moment arm.
- Unit is N.m

Theorem of Varignon:

The moment of the resultant of two concurrent forces with respect to a centre in their plane is equal to the alzebric sum of the moments of the components with respect to some centre.

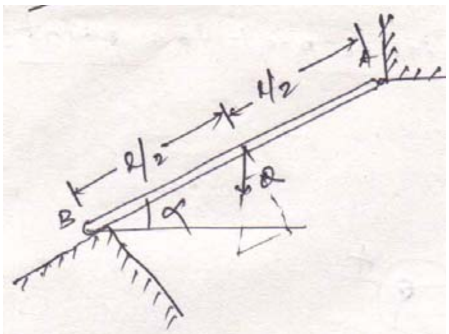
Problem 1:

A prismatic clear of AB of length l is hinged at A and supported at B. Neglecting friction, determine the reaction R_b produced at B owing to the weight Q of the bar.

Taking moment about point A,

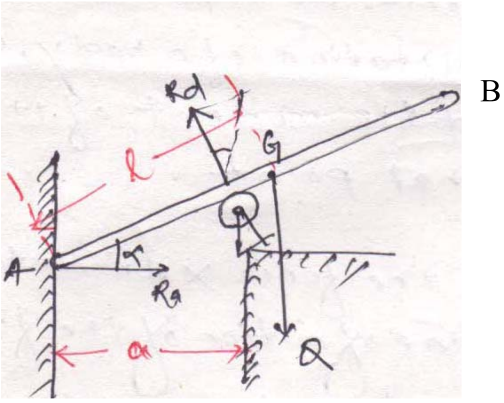
$$R_b \times l = Q\cos\alpha.\frac{l}{2}$$

$$\Rightarrow R_b = \frac{Q}{2}\cos\alpha$$



Problem 2:

A bar AB of weight Q and length 2l rests on a very small friction less roller at D and against a smooth vertical wall at A. Find the angle α that the bar must make with the horizontal in equilibrium.



Resolving vertically,

$$R_d \cos\alpha = Q$$

Now taking moment about A,

$$\frac{R_d . a}{\cos\alpha} - Q.l\cos\alpha = 0$$

$$\Rightarrow \frac{Q.a}{\cos^2\alpha} - Q.l\cos\alpha = 0$$

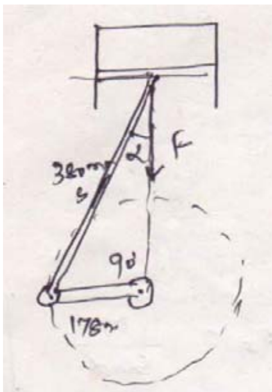
$$\Rightarrow Q.a - Q.l\cos^3\alpha = 0$$

$$\Rightarrow \cos^3\alpha = \frac{Q.a}{Q.l}$$

$$\Rightarrow \alpha = \cos^{-1}\sqrt[3]{\frac{a}{l}}$$

Problem 3:

If the piston of the engine has a diameter of 101.6 mm and the gas pressure in the cylinder is 0.69 MPa. Calculate the turning moment M exerted on the crankshaft for the particular configuration.



Area of cylinder

$$A = \frac{\pi}{4}(0.1016)^2 = 8.107 \times 10^{-3} m^2$$

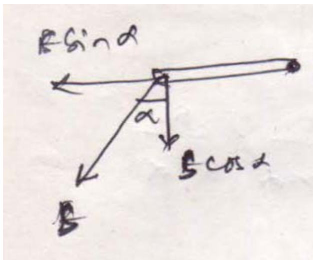
Force exerted on connecting rod,

F = Pressure × Area
 = $0.69 \times 10^6 \times 8.107 \times 10^{-3}$
 = 5593.83 N

$$\text{Now } \alpha = \sin^{-1}\left(\frac{178}{380}\right) = 27.93^\circ$$

$$S\cos\alpha = F$$

$$\Rightarrow S = \frac{F}{\cos\alpha} = 6331.29N$$

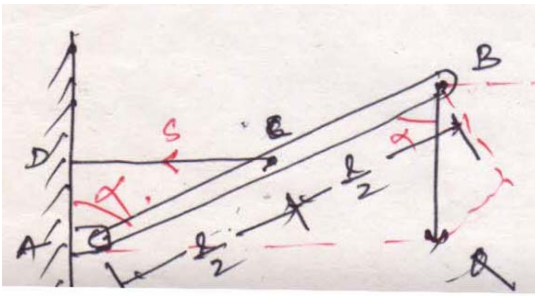


Now moment entered on crankshaft,

$$S\cos\alpha \times 0.178 = 995.7N = 1KN$$

Problem 4:

A rigid bar AB is supported in a vertical plane and carrying a load Q at its free end. Neglecting the weight of bar, find the magnitude of tensile force S in the horizontal string CD.



Taking moment about A,

$$\sum M_A = 0$$

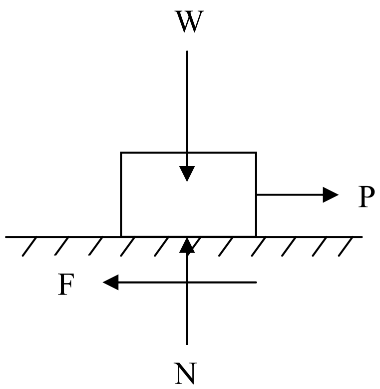
$$S.\frac{l}{2}\cos\alpha = Q.l\sin\alpha$$

$$\Rightarrow S = \frac{Q.l\sin\alpha}{\frac{l}{2}\cos\alpha}$$

$$\Rightarrow S = 2Q.\tan\alpha$$

Friction

- The force which opposes the movement or the tendency of movement is called **Frictional force or simply friction**. It is due to the resistance to motion offered by minutely projecting particles at the contact surfaces. However, there is a limit beyond which the magnitude of this force cannot increase.
- If the applied force is more than this limit, there will be movement of one body over the other. This limiting value of frictional force when the motion is impending, it is known as **Limiting Friction**.
- When the applied force is less than the limiting friction, the body remains at rest and such frictional force is called **Static Friction**, which will be having any value between zero and the limiting friction.
- If the value of applied force exceeds the limiting friction, the body starts moving over the other body and the frictional resistance experienced by the body while moving is known as **Dynamic Friction**. Dynamic friction is less than limiting friction.
- Dynamic friction is classified into following two types:
    a) Sliding friction
    b) Rolling friction
- Sliding friction is the friction experienced by a body when it slides over the other body.
- Rolling friction is the friction experienced by a body when it rolls over a surface.
- It is experimentally found that the magnitude of limiting friction bears a constant ratio to the normal reaction between two surfaces and this ratio is called **Coefficient of Friction**.



$$\text{Coefficient of friction} = \frac{F}{N}$$

where F is limiting friction and N is normal reaction between the contact surfaces.

Coefficient of friction is denoted by μ.

Thus, $\mu = \frac{F}{N}$

Laws of friction

1. The force of friction always acts in a direction opposite to that in which body tends to move.
2. Till the limiting value is reached, the magnitude of friction is exactly equal to the force which tends to move the body.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces of contact and this ratio is called coefficient of friction.
4. The force of friction depends upon the roughness/smoothness of the surfaces.
5. The force of friction is independent of the area of contact between the two surfaces.
6. After the body starts moving, the dynamic friction comes into play, the magnitude of which is less than that of limiting friction and it bears a constant ratio with normal force. This ratio is called **coefficient of dynamic friction**.

Angle of friction

Consider the block shown in figure resting on a horizontal surface and subjected to horizontal pull P. Let F be the frictional force developed and N the normal reaction. Thus, at contact surface the reactions are F and N. They can be graphically combined to get the reaction R which acts at angle θ to normal reaction. This angle θ called the angle of friction is given by

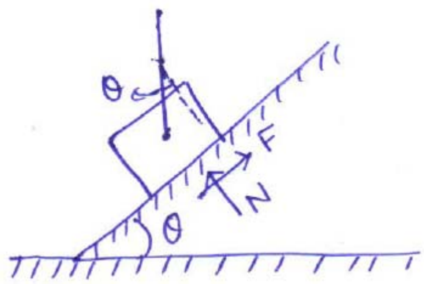
$$\tan\theta = \frac{F}{N}$$

As P increases, F increases and hence θ also increases. θ can reach the maximum value α when F reaches limiting value. At this stage,

$$\tan\alpha = \frac{F}{N} = \mu$$

This value of α is called Angle of Limiting Friction. Hence, the angle of limiting friction may be defined as the angle between the resultant reaction and the normal to the plane on which the motion of the body is impending.

Angle of repose



Consider the block of weight W resting on an inclined plane which makes an angle θ with the horizontal. When θ is small, the block will rest on the plane. If θ is gradually increased, a stage is reached at which the block start sliding down the plane. The angle θ for which the motion is impending, is called the angle of repose. Thus, the maximum inclination of the plane on which a body, free from external forces, can repose is called **Angle of Repose**.

Resolving vertically,
$N = W. \cos\theta$

Resolving horizontally,
$F = W. \sin\theta$

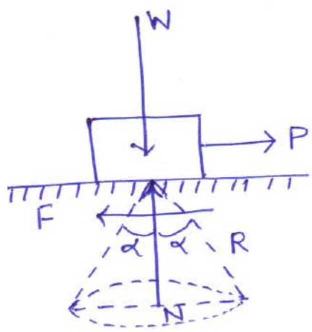
Thus, $\tan\theta = \frac{F}{N}$

If ϕ is the value of θ when the motion is impending, the frictional force will be limiting friction and hence,

$$\tan\phi = \frac{F}{N}$$
$$= \mu = \tan\alpha$$
$$\Rightarrow \phi = \alpha$$

Thus, the value of angle of repose is same as the value of limiting angle of repose.

Cone of friction



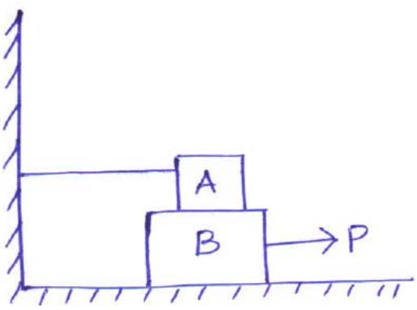
- When a body is having impending motion in the direction of force P, the frictional force will be limiting friction and the resultant reaction R will make limiting angle α with the normal.
- If the body is having impending motion in some other direction, the resultant reaction makes limiting frictional angle α with the normal to that direction. Thus, when the direction of force P is gradually changed through 360˚, the resultant R generates a right circular cone with semi-central angle equal to α.

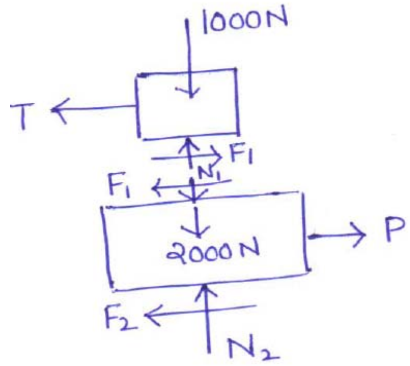
Problem 1: Block A weighing 1000N rests over block B which weighs 2000N as shown in figure. Block A is tied to wall with a horizontal string. If the coefficient of friction between blocks A and B is 0.25 and between B and floor is 1/3, what should be the value of P to move the block (B), if

(a) P is horizontal.

(b) P acts at 30° upwards to horizontal.

Solution: (a)





Considering block A,

$$\sum V = 0$$

$$N_1 = 1000N$$

Since F_1 is limiting friction,

$$\frac{F_1}{N_1} = \mu = 0.25$$

$$F_1 = 0.25N_1 = 0.25 \times 1000 = 250N$$

$$\sum H = 0$$

$$F_1 - T = 0$$

$$T = F_1 = 250N$$

Considering equilibrium of block B,

$$\sum V = 0$$

$$N_2 - 2000 - N_1 = 0$$

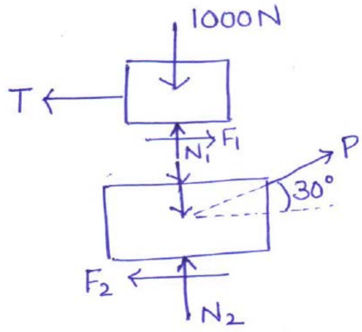
$$N_2 = 2000 + N_1 = 2000 + 1000 = 3000N$$

$$\frac{F_2}{N_2} = \mu = \frac{1}{3}$$

$$F_2 = 0.3N_2 = 0.3 \times 1000 = 1000N$$

$$\sum H = 0$$
$$P = F_1 + F_2 = 250 + 1000 = 1250N$$

(b) When P is inclined:



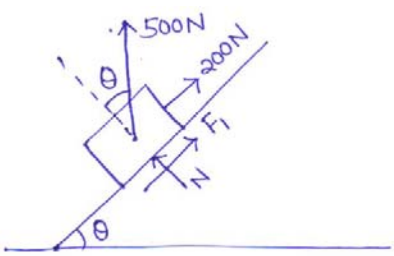
$$\sum V = 0$$
$$N_2 - 2000 - N_1 + P.\sin 30 = 0$$
$$\Rightarrow N_2 + 0.5P = 2000 + 1000$$
$$\Rightarrow N_2 = 3000 - 0.5P$$

From law of friction,

$$F_2 = \frac{1}{3}N_2 = \frac{1}{3}(3000 - 0.5P) = 1000 - \frac{0.5}{3}P$$

$$\sum H = 0$$
$$P\cos 30 = F_1 + F_2$$
$$\Rightarrow P\cos 30 = 250 + \left(1000 - \frac{0.5}{3}P\right)$$
$$\Rightarrow P\left(\cos 30 + \frac{0.5}{3}P\right) = 1250$$
$$\Rightarrow P = 1210.43N$$

Problem 2: A block weighing 500N just starts moving down a rough inclined plane when supported by a force of 200N acting parallel to the plane in upward direction. The same block is on the verge of moving up the plane when pulled by a force of 300N acting parallel to the plane. Find the inclination of the plane and coefficient of friction between the inclined plane and the block.

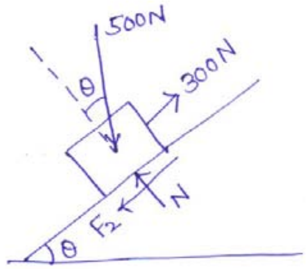


$$\sum V = 0$$
$$N = 500.\cos\theta$$
$$F_1 = \mu N = \mu.500\cos\theta$$

$$\sum H = 0$$

$$200 + F_1 = 500.\sin\theta \quad (1)$$

$$\Rightarrow 200 + \mu.500\cos\theta = 500.\sin\theta$$



$$\sum V = 0$$

$$N = 500.\cos\theta$$

$$F_2 = \mu N = \mu.500.\cos\theta$$

$$\sum H = 0$$

$$500\sin\theta + F_2 = 300 \quad (2)$$

$$\Rightarrow 500\sin\theta + \mu.500\cos\theta = 300$$

Adding Eqs. (1) and (2), we get

$500 = 1000.\ \sin\theta$

$\sin\theta = 0.5$

$\theta = 30^{\circ}$

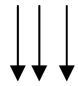
Substituting the value of θ in Eq. 2,

$$500\sin 30 + \mu.500\cos 30 = 300$$

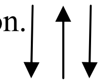
$$\mu = \frac{50}{500\cos 30} = 0.11547$$

Parallel forces on a plane

Like parallel forces: Coplanar parallel forces when act in the same direction.

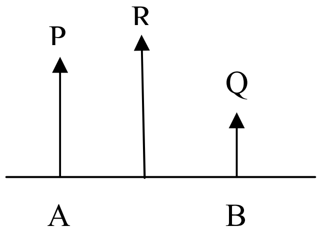


Unlike parallel forces: Coplanar parallel forces when act in different direction.



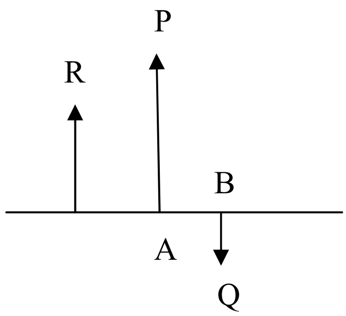
Resultant of like parallel forces:

Let P and Q are two like parallel forces act at points A and B.
R = P + Q



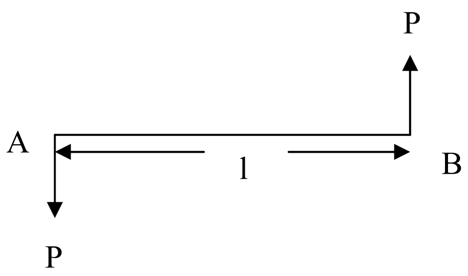
Resultant of unlike parallel forces:
R = P - Q

R is in the direction of the force having greater magnitude.



Couple:

Two unlike equal parallel forces form a couple.

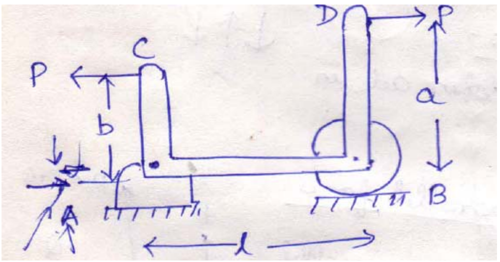


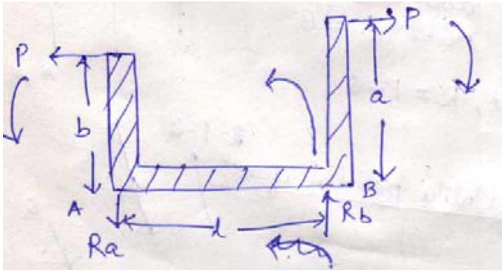
The rotational effect of a couple is measured by its moment.

Moment = P × l

Sign convention: Anticlockwise couple (Positive)
Clockwise couple (Negative)

Problem 1 : A rigid bar CABD supported as shown in figure is acted upon by two equal horizontal forces P applied at C and D. Calculate the reactions that will be induced at the points of support. Assume l = 1.2 m, a = 0.9 m, b =0.6 m.





$$\sum V = 0$$

$$R_a = R_b$$

Taking moment about A,

$$R_a = R_b$$

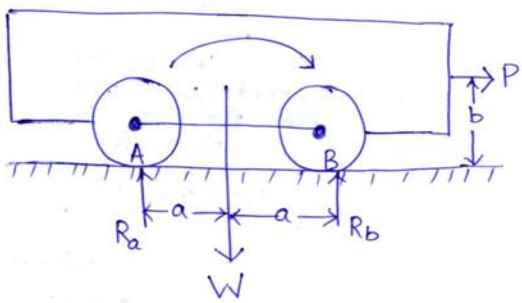
$$R_b \times l + P \times b = P \times a$$

$$\Rightarrow R_b = \frac{P(0.9 - 0.6)}{1.2}$$

$$\Rightarrow R_b = 0.25P(\uparrow)$$

$$\Rightarrow R_a = 0.25P(\downarrow)$$

Problem 2: Owing to weight W of the locomotive shown in figure, the reactions at the two points of support A and B will each be equal to W/2. When the locomotive is pulling the train and the drawbar pull P is just equal to the total friction at the points of contact A and B, determine the magnitudes of the vertical reactions R_a and R_b.



$$\sum V = 0$$

$$R_a + R_b = W$$

Taking moment about B,

$$\sum M_B = 0$$

$$R_a \times 2a + P \times b = W \times a$$

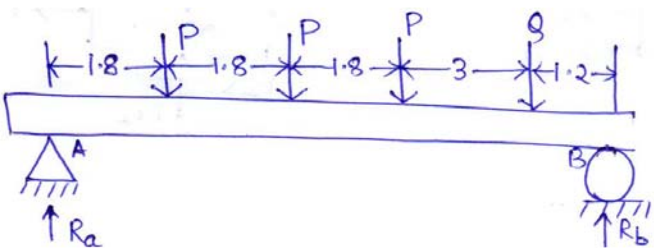
$$\Rightarrow R_a = \frac{W.a - P.b}{2a}$$

$$\therefore R_b = W - R_a$$

$$\Rightarrow R_b = W - \left(\frac{W.a - P.b}{2a}\right)$$

$$\Rightarrow R_b = \frac{W.a + P.b}{2a}$$

Problem 3: The four wheels of a locomotive produce vertical forces on the horizontal girder AB. Determine the reactions R_a and R_b at the supports if the loads P = 90 KN each and Q = 72 KN (All dimensions are in m).



$$\sum V = 0$$

$$R_a + R_b = 3P + Q$$

$$\Rightarrow R_a + R_b = 3 \times 90 + 72$$

$$\Rightarrow R_a + R_b = 342KN$$

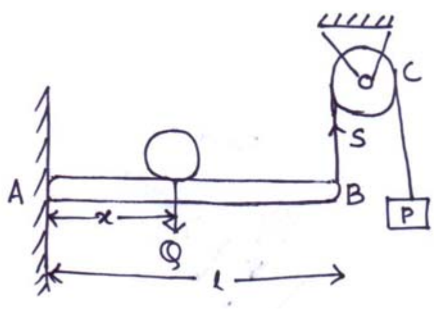
$$\sum M_A = 0$$

$$R_b \times 9.6 = 90 \times 1.8 + 90 \times 3.6 + 90 \times 5.4 + 72 \times 8.4$$

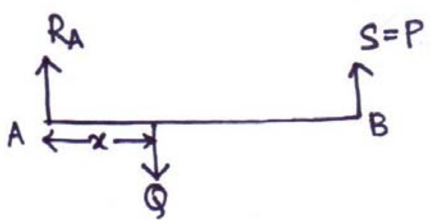
$$\Rightarrow R_b = 164.25KN$$

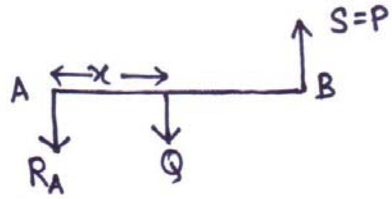
$$\therefore R_a = 177.75KN$$

Problem 4: The beam AB in figure is hinged at A and supported at B by a vertical cord which passes over a frictionless pulley at C and carries at its end a load P. Determine the distance x from A at which a load Q must be placed on the beam if it is to remain in equilibrium in a horizontal position. Neglect the weight of the beam.



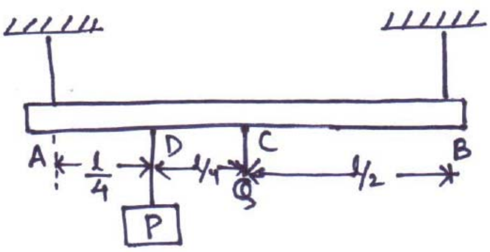
FBD



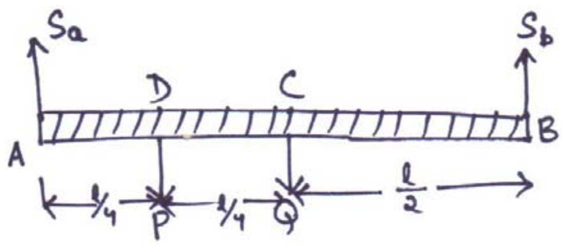


$$\sum M_A = 0$$
$$S \times l = Q \times x$$
$$\Rightarrow x = \frac{P.l}{Q}$$

Problem 5: A prismatic bar AB of weight Q = 44.5 N is supported by two vertical wires at its ends and carries at D a load P = 89 N as shown in figure. Determine the forces S_a and S_b in the two wires.



Q = 44.5 N
P = 89 N



Resolving vertically,

$$\sum V = 0$$
$$S_a + S_b = P + Q$$
$$\Rightarrow S_a + S_b = 89 + 44.5$$
$$\Rightarrow S_a + S_b = 133.5N$$

$$\sum M_A = 0$$
$$S_b \times l = P \times \frac{l}{4} + Q \times \frac{l}{2}$$
$$\Rightarrow S_b = \frac{P}{4} + \frac{Q}{2}$$
$$\Rightarrow S_b = \frac{89}{4} + \frac{44.5}{2}$$
$$\Rightarrow S_b = 44.5$$
$$\therefore S_a = 133.5 - 44.5$$
$$\Rightarrow S_a = 89N$$

Centre of gravity

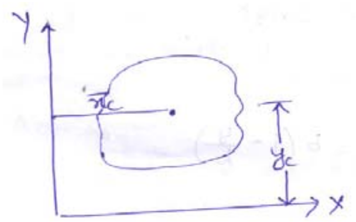
Centre of gravity: It is that point through which the resultant of the distributed gravity force passes regardless of the orientation of the body in space.

- As the point through which resultant of force of gravity (weight) of the body acts.

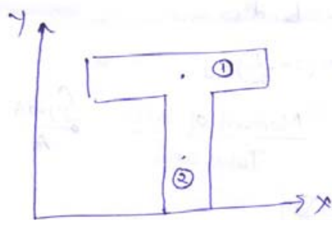
Centroid: Centrroid of an area lies on the axis of symmetry if it exits.

Centre of gravity is applied to bodies with mass and weight and centroid is applied to plane areas.

$$x_c = \sum A_i x_i$$
$$y_c = \sum A_i y_i$$

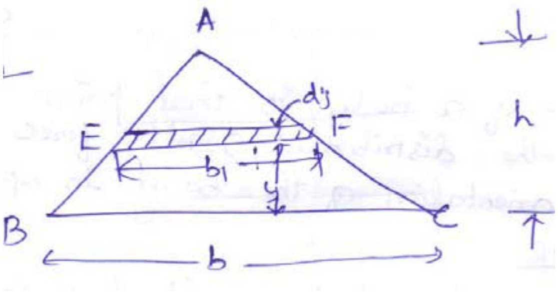


$$x_c = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$
$$y_c = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$



$$x_c = y_c = \frac{\text{Moment of area}}{\text{Total area}}$$
$$x_c = \frac{\int x.dA}{A}$$
$$y_c = \frac{\int y.dA}{A}$$

Problem 1: Consider the triangle ABC of base 'b' and height 'h'. Determine the distance of centroid from the base.



Let us consider an elemental strip of width 'b_1' and thickness 'dy'.

$$\Delta AEF \sim \Delta ABC$$

$$\therefore \frac{b_1}{b} = \frac{h-y}{h}$$

$$\Rightarrow b_1 = b\left(\frac{h-y}{h}\right)$$

$$\Rightarrow b_1 = b\left(1-\frac{y}{h}\right)$$

Area of element EF (dA) = $b_1 \times dy$

$$= b\left(1-\frac{y}{h}\right)dy$$

$$y_c = \frac{\int y.dA}{A}$$

$$= \frac{\int_0^h yb\left(1-\frac{y}{h}\right)dy}{\frac{1}{2}b.h}$$

$$= \frac{b\left[\frac{y^2}{2}-\frac{y^3}{3h}\right]_0^h}{\frac{1}{2}b.h}$$

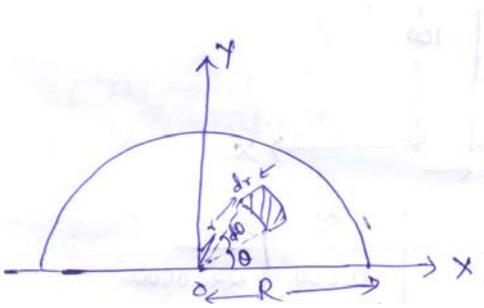
$$= \frac{2}{h}\left[\frac{h^2}{2}-\frac{h^3}{3}\right]$$

$$= \frac{2}{h}\times\frac{h^2}{6}$$

$$= \frac{h}{3}$$

Therefore, y_c is at a distance of h/3 from base.

Problem 2: Consider a semi-circle of radius R. Determine its distance from diametral axis.



Due to symmetry, centroid 'y_c' must lie on Y-axis.

Consider an element at a distance 'r' from centre 'o' of the semicircle with radial width dr.

Area of element = (r.dθ)×dr

Moment of area about x = $\int y.dA$

$$= \int_0^{\pi}\int_0^{R} (r.d\theta).dr \times (r.\sin\theta)$$

$$= \int_0^{\pi}\int_0^{R} r^2 \sin\theta.dr.d\theta$$

$$= \int_0^{\pi}\int_0^{R} (r^2.dr).\sin\theta.d\theta$$

$$= \int_0^{\pi}\left[\frac{r^3}{3}\right]_0^R .\sin\theta.d\theta$$

$$= \int_0^{\pi} \frac{R^3}{3}.\sin\theta.d\theta$$

$$= \frac{R^3}{3}[-\cos\theta]_0^{\pi}$$

$$= \frac{R^3}{3}[1+1]$$

$$= \frac{2}{3}R^3$$

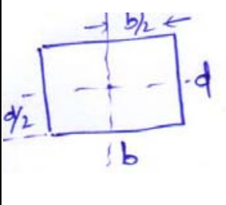
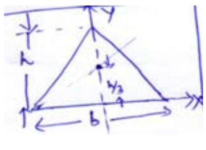
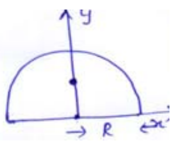
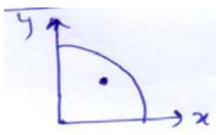
$$y_c = \frac{\text{Moment of area}}{\text{Total area}}$$

$$= \frac{\frac{2}{3} R^3}{\frac{\pi R^2}{2}}$$

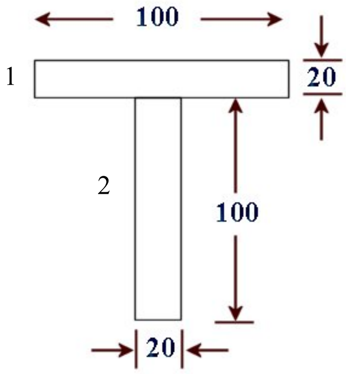
$$= \frac{4R}{3\pi}$$

Therefore, the centroid of the semicircle is at a distance of $\frac{4R}{3\pi}$ from the diametric axis.

Centroids of different figures

Shape	Figure	$\overline{x}$	$\overline{y}$	Area
Rectangle		$\frac{b}{2}$	$\frac{d}{2}$	bd
Triangle		0	$\frac{h}{3}$	$\frac{bh}{2}$
Semicircle		0	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter circle		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{4}$

Problem 3: Find the centroid of the T-section as shown in figure from the bottom.

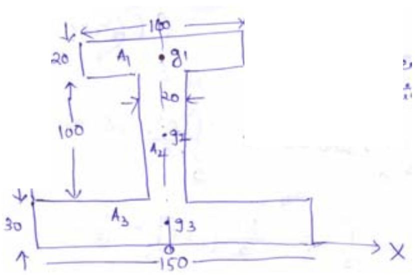


Area (A_i)	x_i	y_i	A_i x_i	A_i y_i
2000	0	110	10,000	22,0000
2000	0	50	10,000	10,0000
4000			20,000	32,0000

$$y_c = \frac{\sum A_i y_i}{A_i} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{32,0000}{4000} = 80$$

Due to symmetry, the centroid lies on Y-axis and it is at distance of 80 mm from the bottom.

Problem 4: Locate the centroid of the I-section.



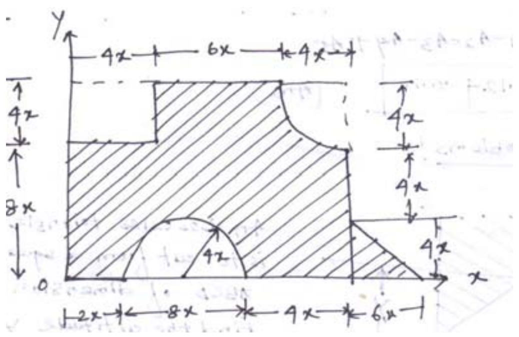
As the figure is symmetric, centroid lies on y-axis. Therefore, $\bar{x} = 0$

Area (A_i)	x_i	y_i	A_i x_i	A_i y_i
2000	0	140	0	280000
2000	0	80	0	160000
4500	0	15	0	67500

$$y_c = \frac{\sum A_i y_i}{A_i} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = 59.71mm$$

Thus, the centroid is on the symmetric axis at a distance 59.71 mm from the bottom.

Problem 5: Determine the centroid of the composite figure about x-y coordinate. Take x = 40 mm.



A_1 = Area of rectangle = 12x.14x=168x^2
A_2 = Area of rectangle to be subtracted = 4x.4x = 16 x^2

A_3 = Area of semicircle to be subtracted = $\dfrac{\pi R^2}{2} = \dfrac{\pi(4x)^2}{2} = 25.13x^2$

A_4 = Area of quatercircle to be subtracted = $\dfrac{\pi R^2}{4} = \dfrac{\pi(4x)^2}{4} = 12.56x^2$

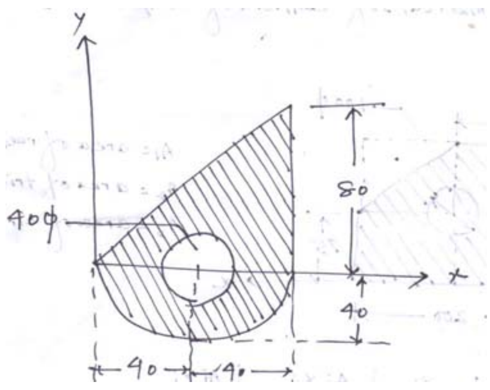
A_5 = Area of triangle = $\dfrac{1}{2} \times 6x \times 4x = 12x^2$

Area (A_i)	**x_i**	**y_i**	**A_i x_i**	**A_i y_i**
A_1 = 268800	7x = 280	6x =240	75264000	64512000
A_2 = 25600	2x = 80	10x=400	2048000	10240000
A_3 = 40208	6x =240	$\dfrac{4\times 4x}{3\pi}$=67.906	9649920	2730364.448
A_4 = 20096	$10x + \left(4x - \dfrac{4\times 4x}{3\pi}\right)$ $= 492.09$	$8x + \left(4x - \dfrac{4\times 4x}{3\pi}\right)$ $= 412.093$	9889040.64	8281420.926
A_5 = 19200	$14x + \dfrac{6x}{3} = 16x$ = 640	$\dfrac{4x}{3} = 53.33$	12288000	1023936

$$x_c = \frac{A_1x_1 - A_2x_2 - A_3x_3 - A_4x_4 + A_5x_5}{A_1 - A_2 - A_3 - A_4 + A_5} = 326.404mm$$

$$y_c = \frac{A_1y_1 - A_2y_2 - A_3y_3 - A_4y_4 + A_5y_5}{A_1 - A_2 - A_3 - A_4 + A_5} = 219.124mm$$

Problem 6: Determine the centroid of the following figure.



A_1 = Area of triangle = $\dfrac{1}{2} \times 80 \times 80 = 3200m^2$

A_2 = Area of semicircle = $\dfrac{\pi d^2}{8} - \dfrac{\pi R^2}{2} = 2513.274m^2$

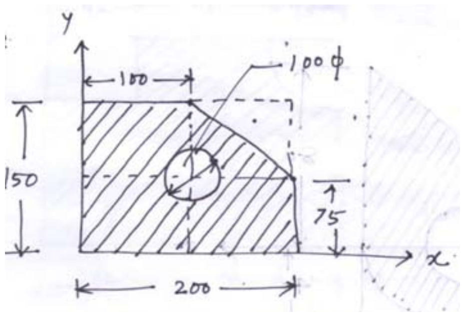
A_3 = Area of semicircle = $\dfrac{\pi D^2}{2} = 1256.64m^2$

Area (A_i)	x_i	y_i	A_i x_i	A_i y_i
3200	2×(80/3)=53.33	80/3 = 26.67	170656	85344
2513.274	40	$\frac{-4\times 40}{3\pi}=-16.97$	100530.96	-42650.259
1256.64	40	0	50265.6	0

$$x_c = \frac{A_1x_1 + A_2x_2 - A_3x_3}{A_1 + A_2 + A_3} = 49.57mm$$

$$y_c = \frac{A_1y_1 + A_2y_2 - A_3y_3}{A_1 + A_2 - A_3} = 9.58mm$$

Problem 7: Determine the centroid of the following figure.



A_1 = Area of the rectangle
A_2 = Area of triangle
A_3 = Area of circle

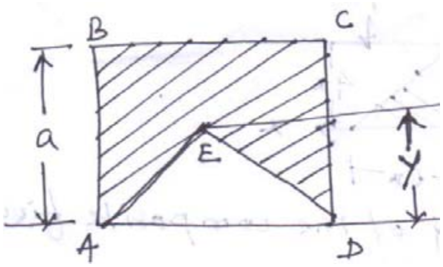
Area (A_i)	x_i	y_i	A_i x_i	A_i y_i
30,000	100	75	3000000	2250000
3750	100+200/3 = 166.67	75+150/3 =125	625012.5	468750
7853.98	100	75	785398	589048.5

$$x_c = \frac{\sum A_ix_i}{\sum A_i} = \frac{A_1x_1 - A_2x_2 - A_3x_3}{A_1 - A_2 - A_3} = 86.4mm$$

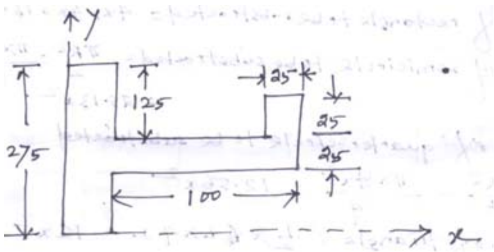
$$y_c = \frac{\sum A_iy_i}{\sum A_i} = \frac{A_1y_1 - A_2y_2 - A_3y_3}{A_1 - A_2 - A_3} = 64.8mm$$

Numerical Problems (Assignment)

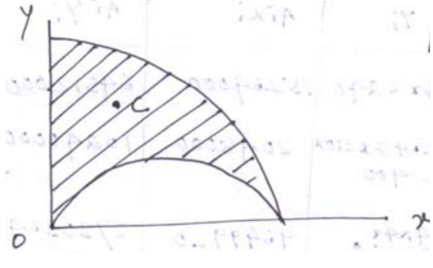
1. An isosceles triangle ADE is to cut from a square ABCD of dimension ‘a’. Find the altitude ‘y’ of the triangle so that vertex E will be centroid of remaining shaded area.



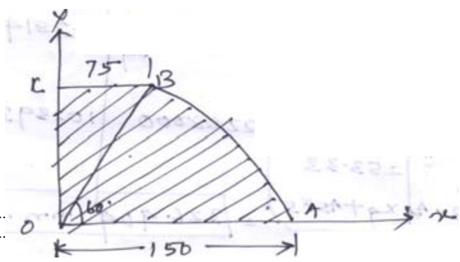
2. Find the centroid of the following figure.



3. Locate the centroid C of the shaded area obtained by cutting a semi-circle of diameter ‘a’ from the quadrant of a circle of radius ‘a’.



4. Locate the centroid of the composite figure.



Module –I

Truss/ Frame: A pin jointed frame is a structure made of slender (cross-sectional dimensions quite small compared to length) members pin connected at ends and capable of taking load at joints.

Such frames are used as roof trusses to support sloping roofs and as bridge trusses to support deck.

Plane frame: A frame in which all members lie in a single plane is called plane frame. They are designed to resist the forces acting in the plane of frame. Roof trusses and bridge trusses are the example of plane frames.

Space frame: If all the members of frame do not lie in a single plane, they are called as space frame. Tripod, transmission towers are the examples of space frames.

Perfect frame: A pin jointed frame which has got just sufficient number of members to resist the loads without undergoing appreciable deformation in shape is called a perfect frame. Triangular frame is the simplest perfect frame and it has 03 joints and 03 members.

It may be observed that to increase one joint in a perfect frame, two more members are required. Hence, the following expression may be written as the relationship between number of joint j, and the number of members m in a perfect frame.

$m = 2j - 3$

(a) When LHS = RHS, Perfect frame.
(b) When LHS<RHS, Deficient frame.
(c) When LHS>RHS, Redundant frame.

Assumptions

The following assumptions are made in the analysis of pin jointed trusses:

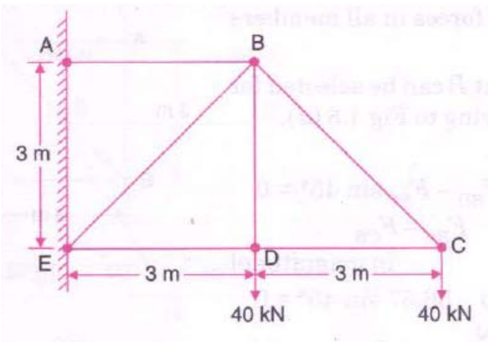
1. The ends of the members are pin jointed (hinged).
2. The loads act only at the joints.
3. Self weight of the members is negligible.

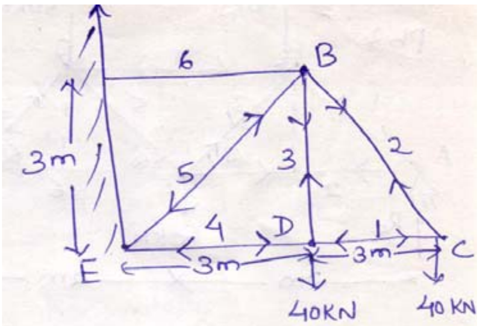
Methods of analysis

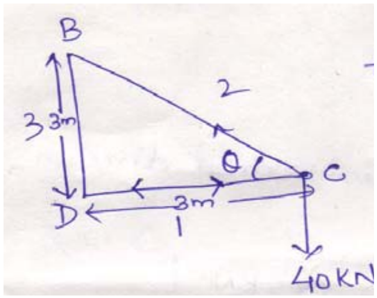
1. Method of joint
2. Method of section

Problems on method of joints

Problem 1: Find the forces in all the members of the truss shown in figure.



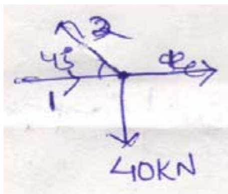




$\tan\theta = 1$

$\Rightarrow \theta = 45^\circ$

Joint C



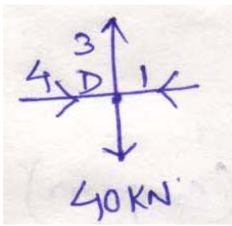
$S_1 = S_2 \cos 45$

$\Rightarrow S_1 = 40KN$ (Compression)

$S_2 \sin 45 = 40$

$\Rightarrow S_2 = 56.56KN$ (Tension)

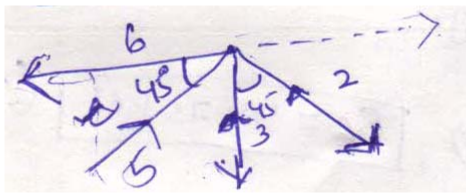
Joint D



$S_3 = 40KN$ (Tension)

$S_1 = S_4 = 40KN$ (Compression)

Joint B



Resolving vertically,

$\sum V = 0$

$S_5 \sin 45 = S_3 + S_2 \sin 45$

$\Rightarrow S_5 = 113.137KN$ (Compression)

Resolving horizontally,

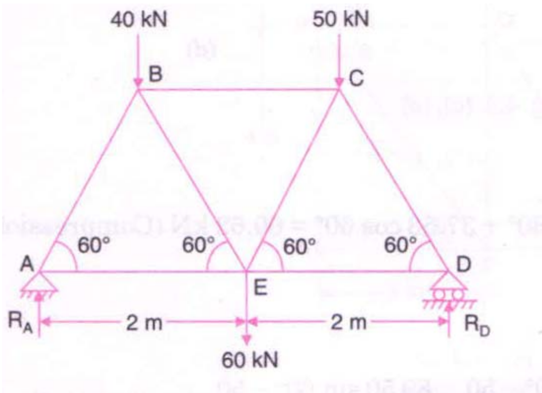
$\sum H = 0$

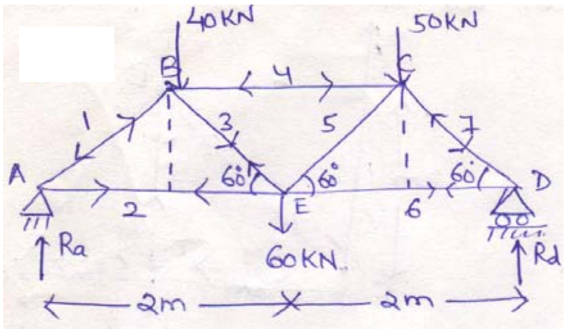
$S_6 = S_5 \cos 45 + S_2 \cos 45$

$\Rightarrow S_6 = 113.137 \cos 45 + 56.56 \cos 45$

$\Rightarrow S_6 = 120KN$ (Tension)

Problem 2: Determine the forces in all the members of the truss shown in figure and indicate the magnitude and nature of the forces on the diagram of the truss. All inclined members are at 60° to horizontal and length of each member is 2m.





Taking moment at point A,

$\sum M_A = 0$

$R_d \times 4 = 40 \times 1 + 60 \times 2 + 50 \times 3$

$\Rightarrow R_d = 77.5KN$

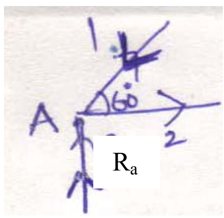
Now resolving all the forces in vertical direction,

$\sum V = 0$

$R_a + R_d = 40 + 60 + 50$

$\Rightarrow R_a = 72.5KN$

<u>Joint A</u>



$\sum V = 0$

$\Rightarrow R_a = S_1 \sin 60$

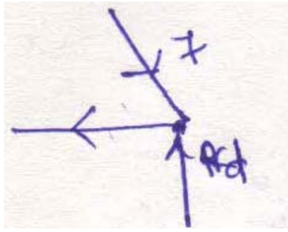
$\Rightarrow S_1 = 83.72KN$ (Compression)

$\sum H = 0$

$\Rightarrow S_2 = S_1 \cos 60$

$\Rightarrow S_1 = 41.86KN$ (Tension)

Joint D



$\sum V = 0$

$S_7 \sin 60 = 77.5$

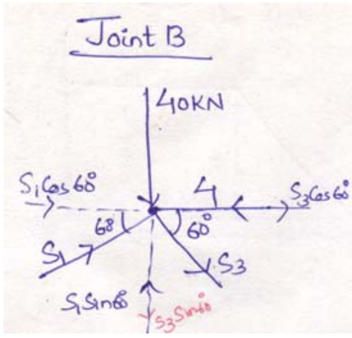
$\Rightarrow S_7 = 89.5KN$ (Compression)

$\sum H = 0$

$S_6 = S_7 \cos 60$

$\Rightarrow S_6 = 44.75KN$ (Tension)

Joint B



$\sum V = 0$

$S_1 \sin 60 = S_3 \cos 60 + 40$

$\Rightarrow S_3 = 37.532KN$ (Tension)

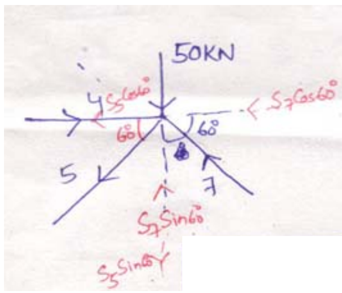
$\sum H = 0$

$S_4 = S_1 \cos 60 + S_3 \cos 60$

$\Rightarrow S_4 = 37.532 \cos 60 + 83.72 \cos 60$

$\Rightarrow S_4 = 60.626KN$ (Compression)

Joint C



$\sum V = 0$

$S_5 \sin 60 + 50 = S_7 \sin 60$

$\Rightarrow S_5 = 31.76KN$ (Tension)